HORIZON CONTENT KNOWLEDGE IN PRESERVICE TEACHER TEXTBOOKS: AN APPLICATION OF NETWORK ANALYSIS

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Horizon Content Knowledge (HCK, Ball, Thames & Phelps, 2008), knowledge that teachers hold about the interconnectedness of mathematics, has been recognized as an integral component of Mathematics Knowledge for Teaching, yet little is known about how or where it is taught in the preservice teacher (PST) curriculum or how researchers should study HCK. This study used network analysis to examine HCK in the context of undergraduate PST textbooks. I describe my approach to the study and report on the analytical questions I encountered. I then describe the affordances and constraints of network analysis for understanding HCK and reflect on the power of this tool to understand HCK in a variety of contexts.

Keywords: Curriculum Analysis, Data Analysis and Statistics, Mathematical Knowledge for Teaching, Teacher Education-Preservice

Ball, Thames, and Phelps (2008) developed a framework for the Mathematical Knowledge for Teaching (MKT) which divides MKT into six subdomains, three of which are oriented around subject matter knowledge (SMK) and three around pedagogical content knowledge (PCK). SMK is further divided into three dimensions: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Horizon Content Knowledge (HCK). CCK is mathematical content that is generally known outside of the world of teaching; SCK is mathematical content that teachers rely on to teach; and HCK is often noted as knowledge teachers hold about the interconnectedness of mathematics. One example of HCK is the knowledge that teachers possess regarding how addition and multiplication are connected mathematical domains.

Little work has been done to examine what HCK is being taught to PSTs. One place to look for answers to these questions is in PST textbooks. Because of the varying definitions of HCK in the literature, and because the interconnectedness of mathematics may be more difficult to define and analyze than CCK or SCK, the study of HCK presents unique analytical challenges that network analysis may address. For this study, I examined three texts for a mathematical content course for elementary PSTs using network analysis to examine HCK in the context of addition of whole numbers. I present my methodological approach to this study, along with reflections on the use of network analysis for examining HCK. I discuss the analytical choices that I encountered and the affordances and constraints of network analysis for understanding HCK in PST textbooks. Finally, I outline how network analysis may be used in future work on HCK.

Theoretical Framework and Purpose of Study

PSTs come to understand disciplinary knowledge within a socio-cultural environment. As Lave and Wenger (2007) note: “This world is socially constituted; objective forms and systems of activity, on the one hand, and agents’ subjective and intersubjective understandings of them on the other, mutually constitute both the world and its experienced forms” (p. 51). Thus, the HCK of individual teachers is the result of a process of socially reproducing culturally and socially situated knowledge. In that process of social reproduction, PST textbooks, as artifacts,

transmit cultural tools the mathematics community values in the education of PSTs (Wertsch, 1998). As such textbooks play an important role in cultivating the HCK of PSTs.

This study, situated in a socio-cultural framing, makes three major assumptions. First that PST textbooks, as cultural artifacts, describe some of the cultural tools PSTs are exposed to during a math content course. Second, that the Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) as a community-generated artifact, represents the domains of CCK that teachers are expected to be familiar with. The final assumption is that I, as a full participant in the mathematical community and the community of teacher educators, understand the cultural tools and norms of this community. This positions me to create a reflection of the HCK that I recognize in PST textbooks. A result of this framing is that the HCK I have identified is one of many representations of HCK that may facilitate PST understanding of the interconnectedness of mathematics. Results about HCK from this study are bounded by the cultural artifacts included in the analysis and by my own knowledge. This is a socially, culturally, and historically situated analysis of HCK from three textbooks.

Though the MKT framework is well-established, there is not full agreement in the literature on what constitutes HCK (Ball & Bass, 2009; Ball, Thames & Phelps, 2008; Figueiras, Ribeiro, Carrilo, Fernández, and Deulofeu, 2011; Zazkis & Mamolo, 2011). Ball, Thames, and Phelps (2008) defined HCK as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403). The following year, Ball and Bass (2009) expanded on this idea and outlined four aspects of HCK: (1) "A sense of the mathematical environment surrounding the current 'location' in instruction"; (2) Major disciplinary ideas and structures"; (3) Key mathematical practices; and (4) Core mathematical values and sensibilities (p. 6). For this study, I operationalize HCK using Ball and Bass's (2009) first and second aspects of HCK and also integrate a similar definition proposed by Figueiras et al. (2011) which defines HCK as "connections between mathematical concepts and ideas, grounded in the coherence of mathematics, in which all concepts and ideas are precisely defined and logically interwoven" (p. 28). Based on these two definitions, I identify eighteen broad mathematical concepts (major disciplinary ideas) present in four cultural artifacts (the CCSSM and three PST texts). I consider HCK present when two or more of those eighteen mathematical concepts are connected by the textbook author in a coherent way (within a single paragraph).

Method

Sampling and Coding

Three textbooks were chosen for this study on the HCK based on a single criterion: the needed to be reasonable to use in a mathematical content course for elementary undergraduate PSTs. The three books selected were: Manes and Holmes (2018); Sowder, Sowder and Nickerson (2017); and Caldwell, Karp, and Bay-Williams (2011). Manes and Holmes (2018) authored an Open Educational Resource written to help students "think like a mathematician," (p. 1) and which emphasized the Common Core Standards for Mathematical Practice (SMP). Similarly centering the SMP, Sowder et al. (2017) aimed to support classrooms where "students take an active part in learning" (p. xi) and provide students insight into "children's mathematical thinking" (p. xi). Caldwell et al. (2011) is a text written solely about addition and subtraction as part of a larger series of texts published by the National Council of Teachers of Mathematics. I consider this to be a high-level text for an undergraduate PST content course and a possible exemplar of HCK. After texts were chosen, I selected a single topic for study: addition of whole

numbers. Sections of the book that explicitly referenced addition of whole numbers were coded and analyzed. Data were chunked into paragraphs, using one paragraph as the base level of analysis. Relying on the definitions by Ball and Bass (2009) and Figueiras et al. (2011), paragraphs were coded as containing HCK if they connected two different mathematical concepts in a coherent picture of mathematics. I did not code paragraphs as containing horizon content knowledge if the text only implicitly relied on previous concepts, as many new mathematical concepts rely on existing mathematical knowledge. For this study, I was interested in the information explicitly presented to students, not what they might intimate from the text. I also chose, because of the exploratory nature of this study, to code two concepts as "connected" without being more specific about the relationship of that connection (Carley, 1993). For this analysis, I make no assumptions about the linearity in mathematical domains. For example, I do not assume counting precedes addition or that addition precedes subtraction. I only document that the textbook describes a "connection" between the two concepts by discussing them in the same paragraph. This choice impacts the resulting analysis and I discuss those limitations below.

The study relied on the CCSSM to identify disciplinary ideas that Ball and Bass's conceptualize. When applied, the CCSSM was problematic for uncovering HCK in the textbook content because of differences in the level of content detail between the CCSSM and the PST textbooks. Once I identified this analytical issue, I developed emerging categories from the PST texts, in light of CCSSM standards. I formalized these emerging categories into new conceptual codes (Saldaña, 2016) and these were used to code the disciplinary ideas that defined HCK. These conceptual codes, developed in the first round of coding, more faithfully described the sort of concepts that were connected in the textbooks. The final eighteen codes arose from the CCSSM in the context of the PST textbooks, and described eighteen different disciplinary ideas that I found to be present in the textbooks and standards. The codes are addition (A), subtraction (S), multiplication (M), division (D), word problems (WP), concrete models (C), decomposing (DEC), properties (P), place value (PV), unknown addend (or the relationship between addition and subtraction; UA), counting (CNT), mental math (MM), equations (EQN), algorithms (ALG), definition of addition (DEF), measurement (MEAS), estimation (EST), and functions (FCN). For consistency, all paragraphs were coded a second time using the conceptual codes. Finally, the conceptual codes were checked against the initial CCSSM codes for alignment.

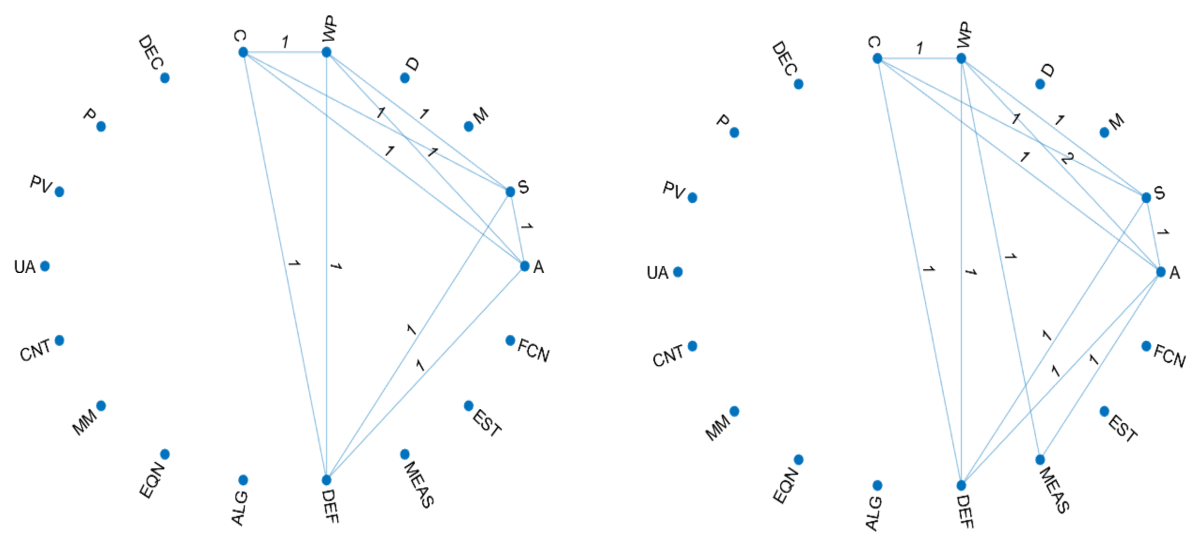
Analysis

Once data was coded, I used network analysis to examine the connections (Borgatti, Mehra, Brass, & Labianca, 2009). Network analysis relies on nodal graphs to visualize connections between objects (in this case, mathematical disciplinary ideas). Borgatti et al. (2009) list several types of analysis that can be conducted using network theory, including an analysis of types of connections, and an analysis of the underlying structure of connected systems. Network analysis has been used in education in a variety of contexts (Carolan, 2013) and network analysis is widely used outside of education to understand connected or relational data (Scott, 2013).

Since network analysis has not been previously used to analyze HCK, I briefly describe how the resulting visualizations originate from the initial coding. Here is an example paragraph from Sowder, Sowder, and Nickerson (2017). This paragraph was coded as containing addition, subtraction, concrete representations (the paragraph references a picture of addition which I omit for space considerations), word problems, and definitions. Codes are noted in brackets. The paragraph mentions "situations" which, in context, is about word problems. A network graph of this paragraph is given on the left side of Figure 1.

> Although the above distinction between meanings for addition [A; DEF] may be subtle, the distinction between meanings for subtraction [S] is quite stark. In Take-from situations [WP], part of a quantity is removed and another part is left over. Such situations [WP] are consistent with the familiar take-away meaning of subtraction [S]. This meaning tends to be emphasized in instruction in the primary grades and tends to be more familiar to students. In Compare situations, by contrast, there is no removal action. Two quantities are simply being compared additively. Children who associate subtraction with take-away may have difficulty solving Compare problems (p. 48).

Each of the eighteen conceptual codes is represented by a node on the graph. The entire network (all nodes) are displayed, even if there are no connections to them. Lines are drawn between nodes that are connected in the paragraph (these are called edges). The five codes from the example paragraph will lead to ten total connections. All codes in the paragraph are connected with edges, so addition is connected to each of the other four topics, subtraction is connected to each of the other four topics, word problems is connected to each of the other four topics, and similarly for concrete representations and definitions. In my analysis, these connections were considered bidirectional, though unidirectional analysis is possible. The number 1 written along the edge is called the edge weight, and it indicates that one paragraph connected these two topics.



Example paragraph   Example and second paragraphs

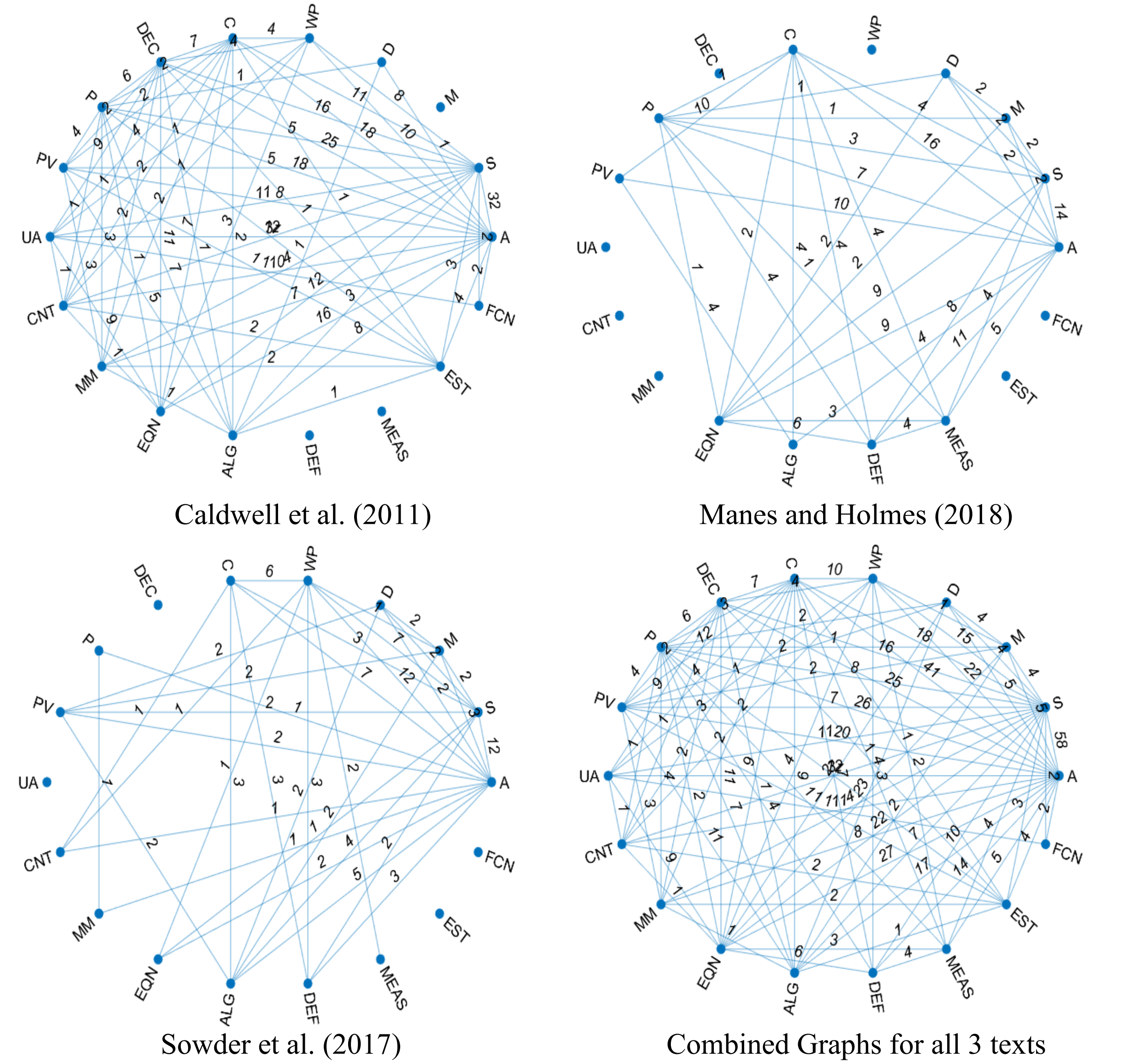
Note. Codes begin with A on the west pole of each graph and progress counter-clockwise
Addition (A), Subtraction (S), Multiplication (M), Division (D), Word Problems (W), Concrete (C), Decomposing (DEC), Properties (P), Place Value (PV), Unknown Addend (UA), Counting (CNT), Mental Math (MM), Equations (EQN), Algorithms (ALG), Definition of Addition (DEF), Measurement (MEAS), Estimation (EST), and Functions (FCN).

Figure 1*: Example Network Analysis Graphs of HCK*

If I add another paragraph coded with addition, word problems, and measurement, the graph changes to look like the right side of Figure 1. The edge weight for addition to word problems now increases to 2 (meaning 2 paragraphs contain this connection) and new edges are added linking addition to measurement and word problems to measurement. Each new paragraph adds edges or adds to the edge weight in the visualization.

Results and Discussion

For HCK, I present a nodal network for each textbook and discuss the results. I consider these results the affordances of using network analysis, namely, the ability to structure and visualize the sorts of HCK textbook authors have discussed. These results are best analyzed as a comparison, so in Figure 2, I provide the three individual graphs for Caldwell et al. (2011), Manes and Holmes (2018) and Sowder et al. (2017), along with a graph that combines their HCK across texts. Looking at Figure 2, Addition and Subtraction are connected in 32 paragraphs in Caldwell et al. (2011). Addition and division are only connected in eight paragraphs, and surprisingly, addition and multiplication are never connected (there is no line between addition



Note. Codes begin with A on the west pole of each graph and progress counter-clockwise
Addition (A), Subtraction (S), Multiplication (M), Division (D), Word Problems (W), Concrete (C), Decomposing (DEC), Properties (P), Place Value (PV), Unknown Addend (UA), Counting (CNT), Mental Math (MM), Equations

(EQN), Algorithms (ALG), Definition of Addition (DEF), Measurement (MEAS), Estimation (EST), and Functions (FCN).

Figure 2: Network Analysis Graphs of HCK

and multiplication). Because this analysis centered on addition of whole numbers, addition is well connected to many topics with significant edge weights, indicating its importance in this network. Subtraction is similarly connected, again with significant weights, as is decomposing indicating that Caldwell et al. routinely linked these concepts. However, division is only linked to three other topics (properties, algorithms and addition) and all with a weight of 1, indicating that Caldwell et al. did not strongly connect division to other topics in the addition section. Topics that were not connected by Caldwell et al. (2011) include multiplication, measurement, and the definition of addition. Caldwell et al. is the only text that connected to functions.

The graph for Manes and Holmes (2018) is less dense than the one for Caldwell indicating fewer paragraphs were coded as containing connections. The graph shows seven disconnected nodes: word problems, decomposing, unknown addend, counting, mental math, estimation and functions. Nodes that connect heavily with addition in Manes and Holmes (2018) are concrete models (16), place value (10), equations (9), and the definition of addition (11). Concrete models are also heavily connected to place value. The graph is less dense for Manes and Holmes than for Caldwell et al., but it is difficult to tell if that is the result of a real difference or because of the disproportionate number of paragraphs coded for Caldwell et al. (2011). Finally, the graph for Sowder et al. (2017) shows even fewer connections. Addition is still heavily connected. Nodes that are completely disconnected are decomposing, unknown addend, estimation, and functions. Word problems connect to eight other topics, which confirms that word problems were heavily emphasized in this text. Thirteen is the highest edge weight here (between word problems and addition) with weights decreasing significantly away from the addition subtraction or word problem nodes. Counting is only related to concrete models, word problems and addition, all with a weight of 1. Finally, the combined graph that visualizes the connections present across texts is useful to analyze what sort of HCK textbook authors were emphasizing across texts. So, if the research question were not comparative, but interested in the sort of HCK knowledge that the community of textbooks authors valued, the combined graph would be the tool to answer that question. I consider decisions to make that knowledge more accessible in the next section where I reflect on my analytical techniques and suggest future work.

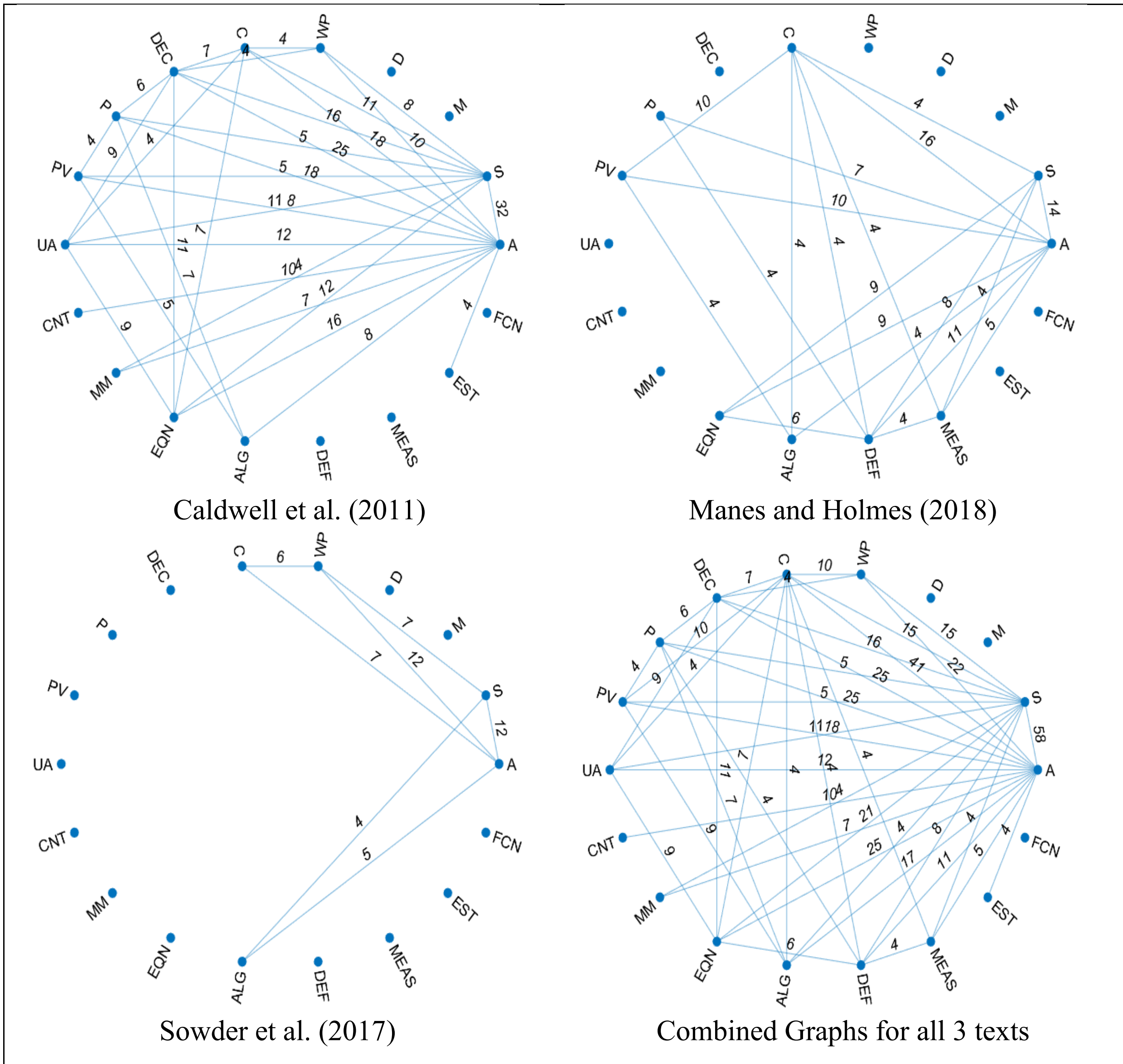
Analytical Questions and Constraints

Operationalizing HCK and Descriptive Validity

This paper presents a fundamentally new approach to analyzing HCK. As such, there are still ongoing analytical questions and decisions that need to be made before utilizing network analysis. I operationalized HCK by claiming that HCK was present if two concepts were present in a single paragraph in a cohesive way, even if the book did not make the exact nature of the connection explicit. For example, my results claim that addition to subtraction HCK is present if addition and subtraction were present in the same paragraph. This operationalization and the resulting analysis raised questions about the descriptive validity of my operationalized variable as compared to the definitions by Ball and Bass (2009) and Figueiras et al. (2011). First, is addition to subtraction HCK "present" in a textbook, if a single paragraph mentions addition and subtraction together? This answer must be no. Single mentions of two topics in one paragraph do not present a "sense of the mathematical environment surrounding the current 'location' in

instruction" (Ball & Bass, 2009, p. 6). However, if a textbook contains 34 paragraphs that all mention "addition and subtraction," does that present a more cohesive picture of mathematical terrain? I believe the answer here is yes.

Given these observations, I propose it may be useful to set a bottom required edge weight. Consider the graphs in Figure 3, which represent the same network as Figure 2, but assume, for discussion, a required bottom edge weight of 4 (meaning, four separate paragraphs need to mention the connection in order to be included in the graph). This bottom edge weight privileges concepts that are mentioned at least four times in the text, which may help to more precisely operationalize HCK in texts. Setting a bottom edge weight also lowers the visual complexity of the figure, allowing the most prominent connections to be easily understood. It highlights both the interconnectedness of the Caldwell text and that most of the connections in Sowder et al. had an edge weight of 1, 2 or 3.



Note. Codes begin with A on the west pole of each graph and progress counter-clockwise

Addition (A), Subtraction (S), Multiplication (M), Division (D), Word Problems (W), Concrete (C), Decomposing (DEC), Properties (P), Place Value (PV), Unknown Addend (UA), Counting (CNT), Mental Math (MM), Equations (EQN), Algorithms (ALG), Definition of Addition (DEF), Measurement (MEAS), Estimation (EST), and Functions (FCN).

Figure 3: Network Analysis Graphs of HCK with Bottom Edge Weight of 4

But other questions arise about how to define and operationalize connections between content. Other types of connectedness in the context of HCK may be present. For example, HCK can refer to both knowledge across grade bands as well as knowledge across domains. It is possible that the four types of HCK (Ball & Bass, 2009) should be coded independently. Other ways to possibly categorize HCK may be: explicit connection made in the text, implicit connection made in the text, across grade band connection, or within grade band connection. It may also be possible to conceptualize an analysis that does rely on a linear conception of these mathematical ideas, for example, using the knowledge from learning trajectories to map the unidirectional connections present (Clements & Sarama, 2004; Confrey et al., 2011). Regardless of the refinement, refining the operationalization of HCK is necessary for more nuanced analysis and a fuller understanding of the content present in texts.

Sampling and Bounding the Analysis

As Scott (2013) noted, inorganic bounding of relational data can lead to false conclusions about the full network. In the context of textbook analysis, bounding is a necessary pragmatic decision. In this study, the sample was bounded to paragraphs that were explicitly focused on addition. Because of this, my network does not faithfully represent the entire network of connections authors made *even for addition* because I did not code every paragraph in the text that mentioned addition. In other words, sampling decisions in network analysis not only impact the edges present in the network, they impact the network nodes as well. For example, perhaps an author does not mention the connection between addition and multiplication in the sections on addition but mentions that connection in the section on multiplication. In that circumstance, and because of the way I have bounded my analysis here, the network I create would obfuscate the actual HCK presentation in the text. If a researcher wanted to create a full network for a text, it is reasonable to expect significant difficulties in generating the network (i.e. the coding scheme) for all content knowledge that PSTs are exposed to across textbooks. Understanding that network analysis of a bounded sample leads to a partial network is essential for interpreting these results.

Conclusion

I have presented what PST texts present as HCK as defined by Ball and Bass (2009), outlining the affordances and constraints of network analysis for HCK and posing some of the analytical questions that require further work and reflection to optimize network analysis for HCK. If these questions can be answered, network analysis can be a significant tool to examine the connectedness of HCK in a variety of situations, not just PST textbooks. A logical next step after textbook analysis is a study of the impact of HCK heavy textbooks on PST knowledge.

The amount and density of information presented in a text for PSTs should be a conscious and strategic choice by textbook authors and teacher educators. There are real differences across populations of students; different teaching environments require different texts. Teacher educators need to be aware of the choices they are making in order to select an appropriate text for their own classes. Further, the teacher education community needs to reach a consensus around what HCK should be included in PST content courses. Taking a careful inventory of what HCK may currently be presented in PST textbooks is one step in that process. An understanding

of HCK is essential to the teacher education community so we can understand what textbooks emphasize, what they leave out and develop consensus in the community about what should be taught in an undergraduate content course for PSTs. Because of the vital role of content courses in PST education, careful attention to HCK in these courses is warranted.

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